On the robustness of association rules

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***Abstract*— Association rules discovery is one of the most important tasks in Knowledge Discovery in Data Bases. The rules produced with APRIORI-like algorithms are then usually used for decision aiding in expert and knowledge based systems and/or by a human end user. Unfortunately such algorithms may produce huge amounts of rules and thus one of the most important steps in association rules discovery is nowadays the evaluation and the interpretation of their interestingness. Objective measures provide numerical information on the quality of a rule and a rule is said "of quality" if its evaluation by a measure is greater than a user defined threshold. In this paper we propose a new specificity of association rule objective interestingness measures: the threshold sensitivity. By dealing with this problem we intend to provide means of measuring the strength/robustness of the interest of a rule. We propose a general framework allowing us to determine the number of examples that a rule can lose while remaining acceptable, for a panel of classical measures that are transformation of the confidence.** ***Keywords*—association rules, robustness**

I. INTRODUCTION

Association rules discovery is one of the most important tasks in Knowledge Discovery in Data Bases. It aims at discovering all frequent patterns among sets of data attributes. The produced rules are then usually used for decision aiding in expert and knowledge based systems and/or by a human end user. Since the initial work presented in [1] and the APRIORI algorithm, many efforts have been done in order to develop efficient algorithms, as in [2], [3], [4], [5], [6] and [7] for example. It is well known that APRIORI-like algorithms may produce huge amounts of rules and thus one of the most important steps in association rules discovery is nowadays the evaluation and interpretation of their interestingness.

Initially, when discovering associations with algorithms such as APRIORI, the interestingness of a rule was measured through its support and its confidence. Unfortunately, measuring the interestingness of rules only within the support and confidence framework is not satisfying. These rules are not necessarily interesting neither from an expert's point of view nor from a statistical one. For example, high confidence should not be confused with high correlation, nor with causality between the antecedent and the consequent of a rule [8].

Thus many interestingness measures have been defined and used in order to find the best rules in a post-processing step. The importance of objective evaluation criteria of interestingness measures has been focused on by [9], [10] and [11]. Interestingness measures have many different qualities or flaws. In order to select good rules one should use a good measure. Depending on the user's objectives, the data mining experts should provide an appropriate interestingness measure in order to filter the huge amount of rules [12]. Interestingness measures have been thoroughly studied in many works: [13], [14], [15], [16], [17], [18], [19], [20] and [21], for instance, have formally extracted several specificities of a large panel of measures. In [22], we propose a decision aid approach aiming at selecting an interestingness measure, according to data and user expectations. This study is based on formal properties of the measures. In [23] we produced an experimental and a formal classification of the measures. These classifications are quite similar and show groups of measures with similar behaviour.

In this paper we propose a new specificity of association rule objective interestingness measures: the threshold sensitivity. Objective measures provide numerical information on the quality of a rule and a rule is said "of quality" if its evaluation by a measure is greater than a user defined threshold. Thus the evaluation of the quality of rules can be sensitive to this threshold according to the measure used. By dealing with this problem we intend to provide means of measuring the strength/robustness of the interest of a rule. To the best of our knowledge this point has not been thoroughly studied in the literature in a simple way.

We propose a general framework allowing us to determine the number of examples that a rule $\mathtt{A} \rightarrow \mathtt{B}$ can lose while remaining acceptable, for a panel of classical measures that are transformation of the confidence.

The paper is organized as follows. We define the problem, our solution/approach in section II. We present the studied measures in section III. These measures are separated in two categories. First the linear transformation of the confidence and second the monotonic transformation of the confidence. In section IV we present our general results for both categories of measures. Then we conclude in section V.

II. Problem

A. Association rule discovery and acceptance threshold

As defined in [1], given a typical market-basket (transactional) database $\mathtt{E}$, an association rule $\mathtt{A} \rightarrow \mathtt{B}$ means that if someone buys the set of items $\mathtt{A}$, then he/she will probably buy the item $\mathtt{B}$. Such sets of items are usually called itemsets. Thus the problem of mining for association rules consists in discovering all the rules that correlate the presence of one itemset with another under minimum support and minimum confidence constraints. The support is the percentage of transactions that contain $\mathtt{A}$ and $\mathtt{B}$, the confidence of $\mathtt{A} \rightarrow \mathtt{B}$ is the ratio of the number of transactions that contain $\mathtt{A}$ and $\mathtt{B}$ against the number of transactions that contain $\mathtt{A}$. The minimum support and confidence values are two thresholds that must be defined by the user.

In a post-processing step, various other measures may be used to finally select interesting rules. Objective measures are said to be data-driven and only take into account the rules cardinalities (p_{ab} the proportion of examples, $p_{a\bar{b}}$ the proportion of counterexamples... see table IV and figure 1), and $\mathtt{n} = |\mathtt{E}|$ the number of transactions.

Let us consider an association rule $\mathtt{A} \rightarrow \mathtt{B}$ whose characteristics are known. Such a rule, when evaluated by an interestingness measure μ is said of interest according to μ if $\mu(\mathtt{A} \rightarrow \mathtt{B}) \geq \sigma_\mu$, where σ_μ has to be fixed by the user. Let us denote by μ_σ^+ the set of such rules.

As an illustrative example let us consider rule $\mathtt{r}_1$ (respectively $\mathtt{r}_2$) whose characteristics in absolute frequencies are given table I (respectively table II) [24]. Their support, confidence and lift values are given table III. Support and confidence are used as a first filter within Apriori algorithms. With such high values the two rules will certainly be generated. Then in a postprocessing step, in order to select the rules the user may use the lift. It is a classical approach. Both rules have the same lift value, greater than 1.0. However do they have the same interestingness? Although having a lower confidence, $\mathtt{r}_1$ may lose 25% of its examples while $\mathtt{r}_2$ may lose only 20% of them (and $\mathtt{r}_2$ have an higher confidence). From this point of view $\mathtt{r}_1$ is more robust when being evaluated in a postprocessing step with the lift and a lift threshold of 1.0.

In this paper we propose a way of evaluating, for linear and monotonic transformation of the confidence, the number of examples a rule $\mathtt{A} \rightarrow \mathtt{B}$ in μ_σ^+ can lose while remaining acceptable.

TABLE I
Relative characteristics of $\mathtt{r}_1$

A\B	0	1	total
0	2000	2000	4000
1	2000	4000	6000
total	4000	6000	10000

TABLE II
Relative characteristics of $\mathtt{r}_2$

A\B	0	1	total
0	1800	3200	5000
1	1000	4000	5000
total	2800	7200	10000

TABLE III
Evaluation of rules r_1 and r_2

	Sup	Conf	Lift
$\mathtt{r}_1$	0.40	0.66	1.11
$\mathtt{r}_2$	0.40	0.80	1.11

B. Strategy

To evaluate the number of examples a rule $\mathtt{A} \rightarrow \mathtt{B}$ in μ_σ^+ can lose while remaining acceptable we proceed as follows:

- a fraction α of the examples is moved towards the counterexamples,
- as showed in table V and figure 2, only p_b is modified, p_a remaining unchanged,
- thus a new rule $\mathtt{A}' \rightarrow \mathtt{B}'$ is obtained, for which $p_{a'} = p_a$, $p_{b'} = p_b - \alpha p_{ab}$, and $p_{a'b'} = p_{ab} - \alpha p_{ab}$

We then evaluate the maximal value of α such that $\mathtt{A}' \rightarrow \mathtt{B}'$ is still an interestingness rule according to σ_μ i.e. such that $\mu(\mathtt{A}' \rightarrow \mathtt{B}') \geq \sigma_\mu$ is still true. Since the measures we will use are decreasing functions of $p_{a\bar{b}}$, this α value is always defined.

TABLE IV
Relative characteristics of $\mathtt{A} \rightarrow \mathtt{B}$

A\B	0	1	total
0	$p_{\bar{a}\bar{b}}$	$p_{\bar{a}b}$	$p_{\bar{a}}$
1	$p_{a\bar{b}}$	p_{ab}	p_a
total	$p_{\bar{b}}$	p_b	1

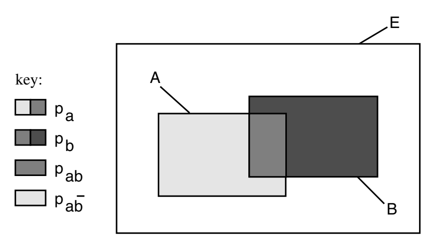

Fig. 1. Relative characteristics of $\mathtt{A} \rightarrow \mathtt{B}$

III. Measures

Objective measures are mathematical functions, defined over the rules' characteristics. Since the cross-classification in table IV has three degrees of freedom, it can entirely be reconstructed using only p_a, p_b and p_{ab}. As association rules focus only on co-occurrences in the data, the quality measures are increasing with respect to p_{ab}, p_a and p_b being fixed.

TABLE V
RELATIVE CHARACTERISTICS OF $\mathtt{A}' \rightarrow \mathtt{B}'$

$\mathtt{A}'\backslash\mathtt{B}'$	0	1	total
0	$p_{\bar{a}\bar{b}}$	$p_{\bar{a}b}$	$p_{\bar{a}}$
1	$p_{a\bar{b}} + \alpha p_{ab}$	$p_{ab} - \alpha p_{ab}$	p_a
total	$p_{\bar{b}} + \alpha p_{ab}$	$p_b - \alpha p_{ab}$	1

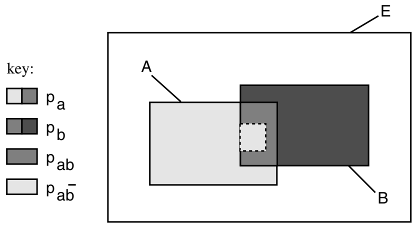

Fig. 2. Relative characteristics of $\mathtt{A}' \rightarrow \mathtt{B}'$

A. Linear transformations of $p_{b/a}$

Amongst the large number of measures defined in the literature, many may be expressed as linear transformations of p_{ab}, and thus of confidence.

Such measures may be written $\mu = \theta_1(\text{Conf} - \theta_0)$.

We distinguish four main kind of linear transformations:

- those where θ_0 and θ_1 are pure constants (Gan)
- those where θ_0 and θ_1 depend on p_a solely (Lap, Sup)
- those where θ_0 and θ_1 depend on p_b solely (CenConf, Lift, Loe)
- those where θ_0 and θ_1 depend on p_a and p_b (LC, PS)

B. Monotonically increasing transformations of $p_{b/a}$

If a measure μ is a monotonically transformation of Conf then we can write $\mu = f(\text{Conf})$. We distinguish three kinds of monotonically increasing transformations of the confidence:

- those not depending on the other characteristics of the rule (ECR and Seb),
- those taking p_b into account (Conv),
- and finally those built on p_a, p_b (Jac).

TABLE VI
LIST OF SELECTED MEASURES

	Name	References
CenConf	centred confidence	
Conf	confidence	[1]
Conv	conviction	[5]
ECR	examples and counter-examples rate	
Gan	Ganascia	[25]
Jac	Jaccard	[26]
Lap	Laplace	[27]
LC	least contradiction	[28]
Lift	Lift	[29]
Loe	Loevinger	[30]
PS	Piatetsky-Shapiro	[9]
Seb	Sebag and Schoenauer	[31]
Sup	support	[1]

TABLE VII
LINEAR TRANSFORMATION OF CONFIDENCE

	Relative definitions	θ_0	θ_1
CenConf	$p_{b/a} - p_b$	p_b	1
Conf	$p_{b/a}$	0	1
Gan	$2p_{b/a} - 1$	0.5	2
Lap	$\frac{p_{b/a} + \frac{1}{np_a}}{1 + \frac{2}{np_a}}$	$-\frac{1}{np_a}$	$\frac{1}{1+\frac{2}{np_a}}$
LC	$2\frac{p_a}{p_b}(p_{b/a} - 0.5)$	0.5	$2\frac{p_a}{p_b}$
Lift	$\frac{p_{b/a}}{p_b}$	0	$\frac{1}{p_b}$
Loe	$\frac{p_{b/a} - p_b}{1 - p_b}$	p_b	$\frac{1}{1-p_b}$
PS	$n(p_{ab} - p_a p_b)$	p_b	np_a
Sup	p_{ab}	0	p_a

TABLE VIII
MONOTONIC TRANSFORMATION OF CONFIDENCE

	Relative definitions	$f(\text{Conf})$
Conv	$\frac{p_a p_{\bar{b}}}{p_{a\bar{b}}}$	$\frac{p_{\bar{b}}}{1-\text{Conf}}$
ECR	$1 - \frac{p_{a\bar{b}}}{p_{ab}}$	$\frac{2\text{Conf}-1}{\text{Conf}}$
Jac	$\frac{p_{ab}}{p_a + p_b - p_{ab}}$	$\frac{\text{Conf}}{1+\frac{p_b}{p_a}-\text{Conf}}$
Seb	$\frac{p_{ab}}{p_{a\bar{b}}}$	$\frac{\text{Conf}}{1-\text{Conf}}$

IV. Results

In this section for a given measure μ, and a given threshold σ, we determine the maximal proportion α of example that a rule may lose, while remaining valid.

A. Support, confidence

The support $\text{Sup}(\mathtt{A} \rightarrow \mathtt{B}) = p_{ab}$ of the initial rule is lowered to $\text{Sup}(\mathtt{A}' \rightarrow \mathtt{B}') = p_{a'b'} = (1-\alpha)p_{ab} = (1-\alpha)\text{Sup}(\mathtt{A} \rightarrow \mathtt{B})$ when a proportion α of the examples are moved to counter-examples. Similarly, we have $\text{Conf}(\mathtt{A}' \rightarrow \mathtt{B}') = (1-\alpha)\text{Conf}(\mathtt{A} \rightarrow \mathtt{B})$.

For $\mathtt{A} \rightarrow \mathtt{B} \in \mu_\sigma^+$, we have the following equivalences:

$$\mathtt{A}' \rightarrow \mathtt{B}' \in \text{Sup}_\sigma^+ \iff \alpha \leq \frac{\text{Sup}(\mathtt{A} \rightarrow \mathtt{B}) - \sigma}{\text{Sup}(\mathtt{A} \rightarrow \mathtt{B})} \quad (1)$$

$$\mathtt{A}' \rightarrow \mathtt{B}' \in \text{Conf}_\sigma^+ \iff \alpha \leq \frac{\text{Conf}(\mathtt{A} \rightarrow \mathtt{B}) - \sigma}{\text{Conf}(\mathtt{A} \rightarrow \mathtt{B})} \quad (2)$$

B. Linear transformations of $p_{b/a}$

Proposition Let $\mathtt{A} \rightarrow \mathtt{B}$ be rule a in μ_σ^+ whose characteristics are as defined in table IV, and $\mathtt{A}' \rightarrow \mathtt{B}'$ a rule derived from $\mathtt{A} \rightarrow \mathtt{B}$ such that its characteristics are as defined in table V. Then for any linear transformation of the confidence $\mu = \theta_1(\text{CONF} - \theta_0)$, $\mathtt{A}' \rightarrow \mathtt{B}'$ is in μ_σ^+ *if and only if:*

$$\alpha \leq \frac{(\mu - \sigma_\mu) + \sigma_\mu(1 - \frac{\theta_1}{\theta_1'})}{\mu + \theta_1\theta_0} + \theta_1 \frac{\theta_0 - \theta_0'}{\mu + \theta_1\theta_0} \qquad (3)$$

where $\mu = \mu(\mathtt{A} \rightarrow \mathtt{B})$, θ_1' and θ_0' are the linear coefficients relative to the new rule $\mathtt{A}' \rightarrow \mathtt{B}'$.

Proof: From $\mu = \theta_1(\text{CONF} - \theta_0)$ we obtain $\text{CONF} = \frac{\mu}{\theta_1} + \theta_0$. Let us denote $\mu(\mathtt{A}' \rightarrow \mathtt{B}')$ by μ' for readability facilities.

$$\begin{aligned} \mu' &= \theta_1'(\text{CONF}' - \theta_0') \\ &= \theta_1'((1-\alpha)\text{CONF} - \theta_0') \\ &= \theta_1'(1-\alpha)(\tfrac{\mu}{\theta_1} + \theta_0) - \theta_1'\theta_0' \\ &= \theta_1'(1-\alpha)\tfrac{\mu}{\theta_1} + \theta_1'\theta_0(1-\alpha) - \theta_1'\theta_0' \end{aligned}$$

$$\mu' \geq \sigma_\mu \Leftrightarrow$$

$$\theta_1'\tfrac{\mu}{\theta_1} + \theta_1'\theta_0 - \theta_1'\theta_0' - \sigma_\mu \geq \alpha(\theta_1'\tfrac{\mu}{\theta_1} + \theta_1'\theta_0) \Leftrightarrow$$

$$\alpha \leq \tfrac{\theta_1'\mu + \theta_1\theta_1'\theta_0 - \theta_1\theta_1'\theta_0' - \sigma_\mu\theta_1}{\theta_1'(\mu + \theta_1\theta_0)} \Leftrightarrow$$

$$\alpha \leq \tfrac{(\mu - \sigma_\mu) + \sigma_\mu(1 - \frac{\theta_1}{\theta_1'})}{\mu + \theta_1\theta_0} + \theta_1\tfrac{\theta_0 - \theta_0'}{\mu + \theta_1\theta_0}$$

This proves inequality 3. ∎

TABLE IX

θ_0' AND θ_1' COEFFICIENTS FOR LINEAR TRANSFORMATIONS OF CONFIDENCE

μ	θ_0'	θ_1'
CENCONF	$p_b - \alpha p_{ab}$	$\theta_1' = \theta_1 = 1$
GAN	$\theta_0' = \theta_0 = 0.5$	$\theta_1' = \theta_1 = 2$
LAP	$\theta_0' = \theta_0 = -\frac{1}{np_a}$	$\theta_1' = \theta_1 = \frac{1}{1+\frac{2}{np_a}}$
LC	$\theta_0' = \theta_0 = 0.5$	$2\frac{p_a}{p_b - \alpha p_{ab}}$
LIFT	$\theta_0' = \theta_0 = 0$	$\frac{1}{p_b - \alpha p_{ab}}$
LOE	$p_b - \alpha p_{ab}$	$\frac{1}{1 - p_b + \alpha p_{ab}}$
PS	$p_b - \alpha p_{ab}$	$\theta_1' = \theta_1 = np_a$
SUP	$\theta_0' = \theta_0 = 0$	$\theta_1' = \theta_1 = p_a$

As an illustration let us consider the centred confidence case.

$\text{CENCONF} = \text{CONF} - p_b$, thus $\theta_1 = 1$ and $\theta_0 = p_b$. When considering $\mathtt{A}' \rightarrow \mathtt{B}'$, p_b is decreased by αp_{ab} as mentioned previously (table V). Thus $\theta_1' = 1$ and $\theta_0' = p_b - \alpha p_{ab}$. It is now easy to see, using inequality 3 that if $\mathtt{A} \rightarrow \mathtt{B}$ is in CENCONF_σ^+ then $\mathtt{A}' \rightarrow \mathtt{B}'$ is also in CENCONF_σ^+ *if and only if:*

$$\alpha \leq \frac{\text{CENCONF} - \sigma}{\text{CENCONF} + p_{\bar{a}b}}$$

Table IX lists the coefficients θ_0' and θ_1' for the 8 linear transformations of the confidence studied in this paper. Using inequality 3 we can deduce the proportion of examples α that a rule in μ_σ^+ may lose while remaining in μ_σ^+ for such a measures. The results are presented in table X.

TABLE X

MAXIMUM VALUES OF α FOR LINEAR TRANSFORMATION OF CONFIDENCE

μ	α_{max}
CENCONF	$\frac{\text{CENCONF} - \sigma}{\text{CENCONF} + p_{\bar{a}b}}$
CONF	$\frac{\text{CONF} - \sigma}{\text{CONF}}$
GAN	$\frac{\text{GAN} - \sigma}{\text{GAN} + 1}$
LAP	$\frac{\text{LAP} - \sigma}{\text{LAP} - \frac{1}{2 + np_a}}$
LC	$\frac{\text{LC}p_b - \sigma p_b}{\text{LC}p_b + p_a - \sigma p_{ab}}$
LIFT	$\frac{\text{LIFT} - \sigma}{\text{LIFT}(1 - \sigma p_a)}$
LOE	$\frac{(\text{LOE} - \sigma)(1 - p_b)}{\text{LOE}(1 - p_b) + p_b - (1 - \sigma)p_{ab}}$
PS	$\frac{\text{PS} - \sigma}{np_{ab}(1 - p_a)}$
SUP	$\frac{\text{SUP} - \sigma}{\text{SUP}}$

C. Monotonically increasing transformations of $p_{b/a}$

For monotonically increasing transformations of $p_{b/a}$we have to consider two cases according whether the measure is depending or not of the characteristics of the rule (except p_{ab}). Let us note $f(\text{CONF})$ these transformations.

If $f(\text{CONF})$ depend on p_{ab} and p_b, or on p_{ab}, p_a and p_b then we have to make a direct calculation from $\mu' = f'(\text{CONF}')$ where f' corresponds to the new rule $\mathtt{A}' \rightarrow \mathtt{B}'$. This is the case for CONV and JAC.

If the measure is not depending of the characteristics of the rule (except p_{ab}), then we have a general result presented inequality 4.

The results are presented in table XI.

Proposition Let $\mathtt{A} \rightarrow \mathtt{B}$ be a rule in μ_σ^+ whose characteristics are as defined in table IV, and $\mathtt{A}' \rightarrow \mathtt{B}'$ a rule derived from $\mathtt{A} \rightarrow \mathtt{B}$ such that its characteristics are as defined in table V. Then for any monotonically increasing transformations f of the confidence depending on p_{ab} solely, $\mathtt{A}' \rightarrow \mathtt{B}'$ is in μ_σ^+ *if and only if:*

$$\alpha \leq \frac{\text{CONF} - f^{-1}(\sigma_\mu)}{\text{CONF}} \qquad (4)$$

where f^{-1} is the reciprocal function of f.

TABLE XI

MAXIMUM VALUES OF α FOR MONOTONIC TRANSFORMATION OF CONFIDENCE

μ	α_{max}
CONV	$\frac{p_a - p_{ab}}{p_{ab}} \frac{\text{CONV} - \sigma}{\sigma - p_a}$
ECR	$\frac{\text{ECR} - \sigma}{2 - \sigma}$
JAC	$1 - \frac{\sigma}{\text{JAC}}$
SEB	$\frac{\text{SEB} - \sigma}{\text{SEB}(1+\sigma)}$

D. Variations of α

For a given measure μ and a given evaluation $\mu(\mathtt{A} \rightarrow \mathtt{B})$ of a rule $\mathtt{A} \rightarrow \mathtt{B}$, table XII shows how the maximum value of α is depending on the rule characteristics (i.e. p_a, p_b and p_{ab}) or not.

For each measure, we evaluate its maximum α, with $\alpha \leq \theta \times [\mu(\mathtt{A} \rightarrow \mathtt{B}) - \sigma]$. The term $\mu(\mathtt{A} \rightarrow \mathtt{B}) - \sigma$ evaluates how far the rule evaluation exceeds the acceptation threshold σ (which depends on the considered μ). This quantity is of course always greater of zero, otherwise the problem does not make sense: the rule is not considered as relevent and thus discarded. Thus, we shall focus on θ variations, with respect to p_a, p_b et p_{ab}.

This leads to different groups measures, depending on their behaviour. The larger group contains CENCONF, CONF, GAN, LAP, PS, CONV and SEB. The second group contains only two measures, LIFT and JAC. All the other measures considered have an individual behaviour. This synthetic overview allows the user to know which characteristic of a rule will make its strength or not, for a given measure.

Two other robustness analysis are immediately possible: an individual rule one, and a group of rules one illustrated below. In the first analysis, our approach immediately gives without extra cost what pourcentage of examples a rule may loose while remaining above the user-defined threshold. This is thus a kind of robustness index. In the second analysis, our approach offers a way of comparing a set of rules.

In order to apprehend the robustness of a rule, two classical thresholds [32], [33] seem interesting to consider: the independance one, for which $p_{ab} = p_a p_b$, and the indetermination one, for which $p_{ab} = p_a/2$.

Let us consider again the two rules r_1 and r_2 presented in table III. They have the same evaluation by LIFT. The presented approach allows the user to select the most robust rule, in our case r_1, although this rule could be seen as dominated by r_2 in a first approach.

In table XIII, r_1 is compared to r_2: they are both evaluated equally by SUP, and r_2 has a higher CONF and LOE value than r_1. Still, r_2 may lose only 20% of its examples while remaining above the independence threshold, while r_1 may loose up to 25% of its examples.

TABLE XII

VARIATIONS OF THE MAXIMUM VALUES OF α WITH p_a, p_b AND p_{ab}

	θ	p_a	p_b	p_{ab}
CENCONF	$\frac{p_a}{p_{ab} - p_a p_{ab}}$	$\nearrow$	$=$	$\searrow$
CONF	$\frac{p_a}{p_{ab}}$	$\nearrow$	$=$	$\searrow$
GAN	$\frac{p_a}{2p_{ab}}$	$\nearrow$	$=$	$\searrow$
LAP	$\frac{2+np_a}{np_{ab}}$	$\nearrow$	$=$	$\searrow$
LC	$\frac{p_b}{p_{ab}(2-\sigma)}$	$=$	$\nearrow$	$\searrow$
LIFT	$\frac{p_a p_b}{p_{ab}(1-\sigma p_a)}$	$\nearrow$	$\nearrow$	$\searrow$
LOE	$\frac{p_a(1-p_b)}{p_{ab}(1-p_a(1-\sigma))}$	$\nearrow$	$\searrow$	$\searrow$
PS	$\frac{1}{np_{ab}(1-p_a)}$	$\nearrow$	$=$	$\searrow$
SUP	$\frac{1}{p_{ab}}$	$=$	$=$	$\searrow$
CONV	$\frac{p_a - p_{ab}}{p_{ab}(\sigma - p_a)}$	$\nearrow$	$=$	$\searrow$
ECR	$\frac{1}{2-\sigma}$	$=$	$=$	$=$
JAC	$\frac{p_a + p_b}{p_{ab}} - 1$	$\nearrow$	$\nearrow$	$\searrow$
SEB	$\frac{p_a - p_{ab}}{p_{ab}(1+\sigma)}$	$\nearrow$	$=$	$\searrow$

TABLE XIII

EVALUATION OF r_1 AND r_2 FOR LOE

	SUP	CONF	LOE	α max
$\mathtt{r}_1$	0.40	0.66	0.16	25%
$\mathtt{r}_2$	0.40	0.80	0.29	20%

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

We propose a general framework allowing us to determine the number of examples that a rule can lose while remaining acceptable. We present results for 13 classical measures that are transformation of the confidence. This allows us to have an objective point of view of the strength/robustness of the interest of a rule. This robustness can then be used as a new criteria for selecting rules. The user can immediately judge if a rule is or not robust according to his/her threshold of rules acceptance. Our solution is very simple to use and does not require extra computational costs. All our calculations are based on rule characteristics and require only very simple arithmetic operations.

B. Future Works

This approach have to be extended to more measures. We decided to move a fraction of the examples towards the counterexamples. Another way of studying the sensitivity could be envisaged with different characteristics of the rules. Our work could be directly used to evaluate the quality of rules in a noisy context. If the sensitivity of a rule is α% and if there is a doubt (or worse a certainty) that data are noisy (with at least α% of noise) then one have to be careful when selecting this rule. We intend to undertake experiments on real databases and to study how many rules are not robust although well rated with classical measures.

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